Special Contributions, Geophysical Institute, Kyoto University, No. 10, 1970, 21-28

Q FACTORS OF THE SCHUMANN RESONANCES AND SOLAR ACTIVITY

By

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(Received November 11, 1970)

Abstract

The vertical components of natural electromagnetic noises of ELF were observed by using a ball antenna, and the power spectra of the earth-ionosphere cavity resonances were produced by using a sonagraph. The peak frequencies and the band widths of 3 db down from the peak were read from over 2000 spectra thus produced in the period from February 1967 to January 1968, and Q factors of the resonances were estimated to be 3.30, 4.52, and 4.92 for the fundamental, second, and third modes respectively. Comparing these values with the previous experimental and theoretical values it is concluded that the greater part of resonant energy is dissipated in the region of ionic conductivity below 56 km. An inverse relation between Q factors and solar activity was found and it is concluded from this that the effective height of the upper boundary of the cavity decreases as the sun becomes active. It is suggested that the sun may have some effect in modifying the conductivity profile below 56 km of the atmosphere.

1. Introduction

It is demonstrated by Schumann [1952 a, b] that the electromagnetic waves resonate in the spherical shell cavity bounded by the homogeneous and isotropic ionosphere and the perfect conductor earth, giving the fundamental mode frequency of 10.6 Hz. Balser and Wagner [1960 a, b] first detected the "Schumann resonances" with a tall tower 37 m in height, while theoretical works of ELF propagation and the Schumann resonances were made notably by Wait [1960], Galejs [1961], and Chapman and Jones [1964]. There are little data accumulated to prove the theories, although some measurements of ELF electric and magnetic components were made by Polk and Fitchen [1962], Chapman and Jones [1964], Gendrin and Stefant [1964], Rycroft and Wormell [1964], and others. In particular, no other experimental values of Q factors were estimated than those obtained by Balser and Wagner [1960 b]. They stated that "the fundamental mode has $Q=4$, and the value increases regularly (although it is difficult to obtain accurate estimates) until for the fifth mode, $Q\approx 6$". As can be understood from the quotation, it is difficult to get a power spectrum of good shape from which Q factors can be estimated. This is because there are static noises at the lower end of this frequency range, which may be caused by irregular movements of space charges in the air near an antenna, and, in addition, at certain times of the day some artificial noises are recorded over the frequency band concerned. The 60 Hz noise from the power line is observed on an antenna at the magnitude of the order of one tenth volt at average places in or near a city, while ELF signals to be observed have an average magnitude of the order of a hundred micro volts; that is, ELF signals are sunk about 60 db below the 60 Hz noise which should be filtered out through an electric circuit.

Q factors of the Schumann resonances are the factors which represent the sharpness of resonances of the earth ionosphere cavity. The earth as the lower boundary of the cavity can be recognized as a perfect conductor, while the ionosphere as an upper boundary is a leaky wall which has finite conductivities. The conductivity distribution of the ionosphere depends on the solar activity because it is mainly produced by the photoionization of solar radiation. Therefore the Q factors of Schumann resonances should have some relation to solar activity. It needs experimental data obtained for a period of at least one year to find a relation between them. It is the purpose of the present investigation to find the relation. In the present paper Q factors are estimated using data for the period of one year from February 1967 to January 1968, although data accumulation is not continuous throughout the year.

2. Observations and data analysis

A ball antenna was used to observe the vertical component of the ELF radio signals at the Geophysical Institute, Kyoto University. The ball antenna is little disturbed by noises such as space charge fluctuations and various artificial noises described above, if we select a suitable site even in populated places (Ogawa *et al.* [1966 a, b]).

Analysis was made with the data obtained throughout the period in most cases from 15th to 21st of each month. ELF signals are recorded on the magnetic tape for about six minutes from 59 minutes to 05 minutes of each hour. In the analysis of the data, signals were reproduced at a speed 40 times faster than the recording speed, and hence the frequencies of the signals were converted from 4.5–45 Hz to 180–1800 Hz. Mean power spectrum was produced each hour by using the sound spectrograph. An example of the spectra thus produced is given in Fig. 1 where the amplitude scale reads 1 db/mm on the original sonagram paper.

The total number of spectra thus produced is 2016 for the entire period.

Peak frequencies f_i and band widths of 3 db down from the peaks Δf_i were estimated from these spectra. Some of the spectra were disturbed by some undesirable natural and artificial noises and such spectra were excluded from the estimation. Thus Q factors were calculated for the first three modes of the Schumann resonances as

$$Q_i = \frac{f_i}{\Delta f_i} \quad (i=1,\ 2,\ 3).$$

The number of spectra used for the final calculations is 1318 for the fundamental mode, 1095 for the second mode and 251 for the third mode. The number of data becomes less as the order of mode increases, because undesirable noises were often recorded superposed at the higher frequencies. The mean values of the Q factors were finally calculated as $Q_1=3.30$, $Q_2=4.52$, and $Q_3=4.95$.

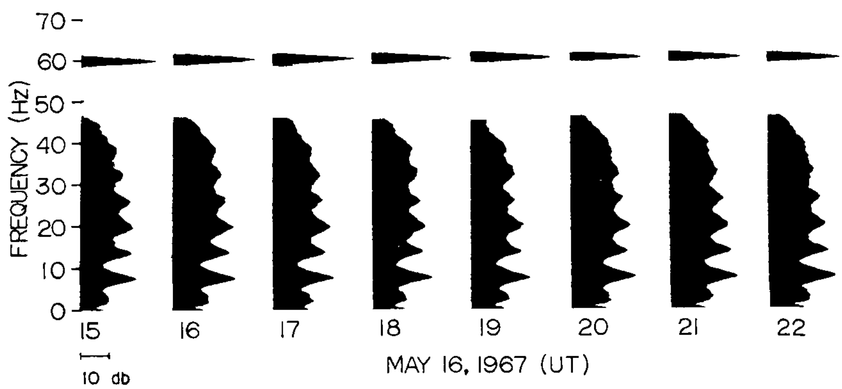

Fig. 1. Examples of power spectra of the Schumann resonances.

3. Q factors and the ionospheric models

Jones [1967] calculated the resonant frequencies and Q factors with various models of the altitude distribution of electric conductivity of the atmosphere up to 100 km. Provided the atmosphere is composed of many thin layers of homogeneous, isotropic and different conductivities he evaluated conditions of reflection and refraction of ELF waves at the each boundary of the layers and calculated the complex root of S_0 of the TM_0 mode equation. The frequency dependent part of the vertical electric field of the Schumann resonance mode which is excited by the electromagnetic waves radiated from the vertical dipole above the ground level, is given by

$$E_r \propto \sum_{n=0}^{\infty} \frac{\omega S_0^2}{\omega_n^2 - \omega^2 S_0^2} \quad (n=0,\ 1,\ 2,\ 3,\ \cdots\cdots), \tag{1}$$

where $\omega_n = 2\pi f_n = c\sqrt{n(n+1)}\,/a$, the eigenfrequencies of the cavity resonances of perfect conductor ionosphere, c the light velocity, a the radius of the earth. From the calculated spectra of E_r given by Eq. (1) the peak frequencies and Q factors can be estimated. The conductivity profiles used are given in Fig. 2 where σ is conductivity and ε_0 permitivity of the space, and the calculated results are listed in Table 1. It is interesting to compare the experimental values with the calculated values. In Fig. 3 are shown the Q factors of the first three modes obtained both from the experiments and the theory. (O) gives the present experimental values and (B) Balser and Wagner's experimental values. (c) gives the values estimated from the model of Cole and Pierce [1965] containing ionic conductivity down to 30 km. (d) gives the theoretical values

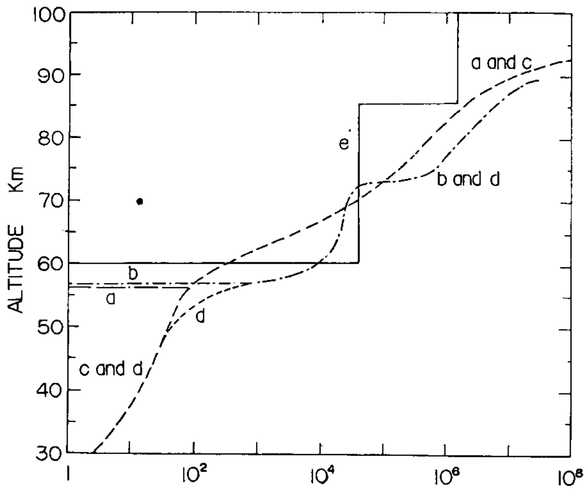

Fig. 2. Atmospheric conductivity profiles used by Jones [1967]. See text for letters.

Table 1. Q factors of the Schumann resonances calculated for various inhomogeneous ionosphere profiles and the observed values (after Jones [1967], except for the present experimental values (O))

Ionosphere profile / Rasonance mode		1	2	3
(a)	Cole and Pierce omitting data below 56 km	12.86	10.82	9.73
(b)	Deeks	9.53	11.2	12.1
(c)	Cole and Pierce	3.83	4.68	5.24
(d)	Deeks and Cole and Pierce	3.50	4.51	5.25
(e)	Two layer profile	4.16	5.20	5.45
(f)	Cole's profile III (Cole's analysis)	4.6	5.1	5.4
Observed values	(B) Balser and Wagner	(4.0)	(4.5)	(5.0)
	(O) Ogawa and Tanaka	3.30	4.52	4.92

estimated from Deeks model with the addition of the (c) model of the levels lower than 56 km. (e) gives the result calculated from the two layer model. (f) gives the values calculated by Cole [1965] with his own model (III).

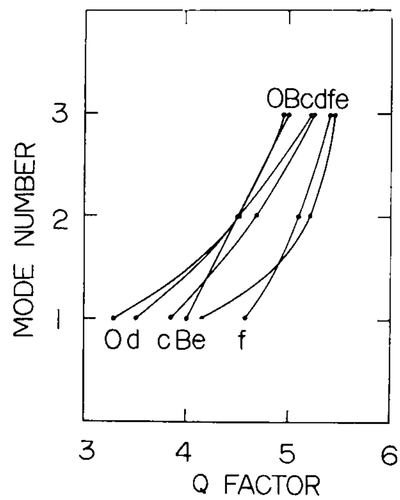

Fig. 3. Comparison of the Q factors of the first three modes of schumann resonances deduced from experimental measurements with the theoretical results by Jones. See text for letters.

The experimental values (O) and (B) have the same values for the second and third modes, while the fundamental mode value of (O) is much smaller than that of (B). It has a value close to the theoretical value of (d). Q factors derived from the conductivity models of (a) and (b) in Jones's paper, which exclude conductivity at levels lower than 56 km, are much higher than the experimental values, while the values considering ionic conductivity below 56 km are rather close to the experimental values. This fact shows that the electromagnetic waves in the Schumann resonance frequency range lost their energy effectively in the region below 56 km. Further, the fact that the experimental results are close to the theoretical results of (c) and (d) shows that an average ionospheric conductivity profile seen globally is close to (c) and (d).

4. Q factors and solar activity

In order to see the correlation between Q factors and solar activity, mean Q factors of each month were compared with the mean values of relative sunspot numbers of corresponding periods. Figs. 4 and 5 give the result of the fundamental and the second modes respectively. There is a clear inverse relation between the Q factors and the relative sunspot numbers for the second mode, although there is a less clear relation of the fundamental mode. The period of data accumulation from February 1967 to January 1968 was a period of increasing solar activity, and considerably large variations of the relative sunspot numbers occurred. It is clear from the figure that when the sun becomes active the Q factor becomes smaller and vice versa.

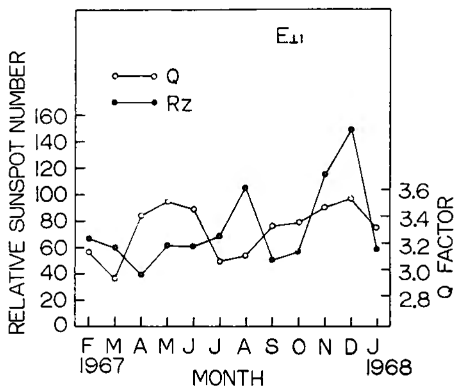

Fig. 4. Correlation between Q factors of the fundamental mode and the relative sunspot numbers.

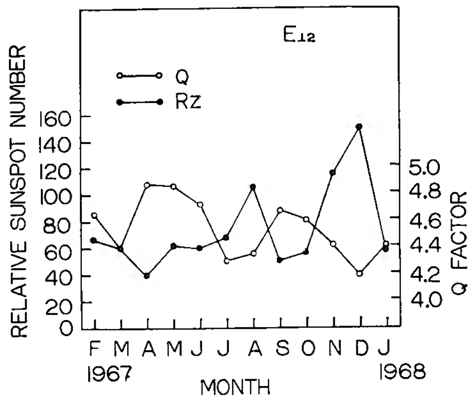

Fig. 5. Correlation between Q factors of the second mode and the relative sunspot numbers.

Following, for example, Atwater [1962] the Q factors of the cavity resonance is expressed by

$$Q=\omega_0\frac{W}{P}, \tag{2}$$

where ω_0 is the resonant angular frequency, W the electromagnetic energy stored in the cavity, and P the energy dissipated in unit time at the resonant angular frequency ω_0. The energy is mainly dissipated at the cavity boundary. W and P are given by

$$W=\frac{\mu}{2}\iiint H^2dV.$$

$$P=\frac{R_s}{2}\iint K^2ds,$$

where μ is the permeability of the space in the cavity, H the magnetic field intensity in the cavity, K the surface current density at the wall, dV the volume element of the cavity, and ds the surface element of the boundary of the cavity. R_s is given by

$$R_s=\frac{\rho}{\delta}=\frac{1}{\delta\sigma}=\sqrt{\frac{\omega_0\mu}{2\sigma}},$$

where ρ is the surface resistivity of the boundary wall, δ the skin depth in the cavity wall and σ the electric conductivity. From the boundary conditions for electromagnetic fields K equals H_t which is the tangential component of the magnetic field. Considering all these factors Eq. (2) may be transformed to

$$Q=\frac{2}{\delta}\frac{\iiint H^2 dV}{\iint H_t^2 ds}.$$

Assuming that $H \approx H_t$, that is, the magnetic field intensity in the dielectric medium filling the cavity equals that in boundary wall, and that the height of the ionospheric boundary from the ground be h, and the volume of the cavity $V \simeq Sh$, then

$$Q=\frac{2}{\delta}\frac{V}{S}=\frac{2h}{\delta}=h\sqrt{2\omega_0\mu\sigma}. \tag{3}$$

From Eq. (3)

$$\frac{dQ}{Q}=\frac{dh}{h}+\frac{1}{2}\frac{d\omega_0}{\omega_0}+\frac{1}{2}\frac{d\sigma}{\sigma}, \tag{4}$$

that is, the rate of change of the Q factor equals the rate of change of the ionospheric height, if other factors remain constant. Alternatively it is equal to half the rate of change of the resonant frequency or the conductivity, if other factors remain constant. It is adequate to explain the inverse relation of Q factors and solar activity by the first term of Eq. (4). As mentioned above, the values of observed Q depend largely upon the conductivity of the atmospheric layer below 56 km, so that the interrelation of Q and solar activity suggests that the sun has an effect on the conductivity profile of this lower region of the atmosphere when the sun becomes active. Neher and Anderson [1962] observed cosmic ray ionization rates in the atmosphere at various atmospheric depths at Thule, Greenland, 88°N geomagnetic latitude, and showed that the observed rates decrease considerably in relation to the solar cycle, depending upon atmospheric depths. This changes the conductivity profile even at lower altitudes, and the effect on Schumann resonance Q factors described here will thus appear.

5. Conclusions

Q factors of the first three modes of the earth ionosphere cavity resonances were estimated from observed natural electromagnetic noises and were compared with the previous experimental values obtaind by Balser and Wagner and with the theoretical values calculated by Jones with various ionospheric conductivity models. As a result the resonant energy of ELF in the earth ionosphere cavity is seen to be mainly dissipated in the upper wall below 56 km of the cavity. Secondly an inverse relation between Q factors and the solar activity was found and it is deduced from this that the sun has an effect in modifying the conductivity profile below 56 km of the atmosphere, and the effective height of the upper boundary of the cavity decreases as the sun becomes active. The relation will be further pursued with an automatic tracking system which is now in work at our laboratory.

One of the other factors that causes time variations of the Q factors is the separation of the observation site from a source region of ELF noises. This effect was shown by Nelson [1967] with simulative computations. In this paper, however, we did not take this effect into consideration and we are preparing similar computations to see the extent of such an effect.

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